$\sin 37^\circ = \frac{3}{5}$	$\sin 53^\circ = \frac{4}{5}$	$\tan 37^\circ = \frac{3}{4}$
$\cos 37^\circ = \frac{4}{5}$	$\cos 53^\circ = \frac{3}{5}$	$\tan 53^\circ = \frac{4}{3}$

Vector

Conditions to be vector :-

1. Should have magnitude as well as direction.
2. Should follow law of vector addition.
3. Addition should be commutative.

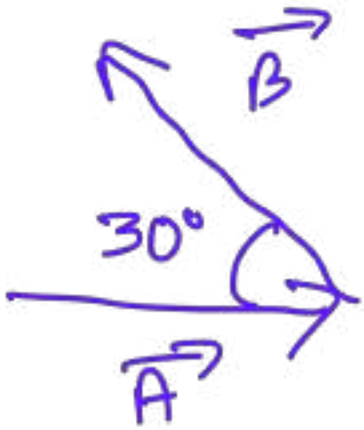
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Eg Small angles are vector but big angles are not.

$\vec{A}$ $\quad$ $\mathbf{A}$ $\quad$ $\bar{A}$

Angle b/w vectors :-

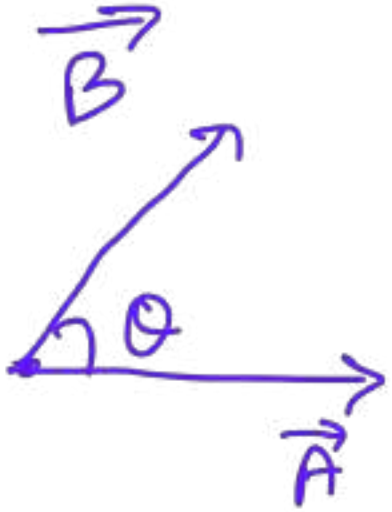
angle b/w vectors is measured when head of both or tail of both touches each other.



$$\theta = 180^\circ - 30^\circ$$
$$= 150^\circ$$

Vector addition :-

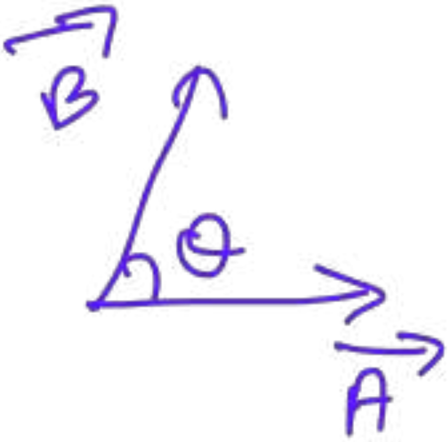
$$\vec{R} = \vec{A} + \vec{B}$$

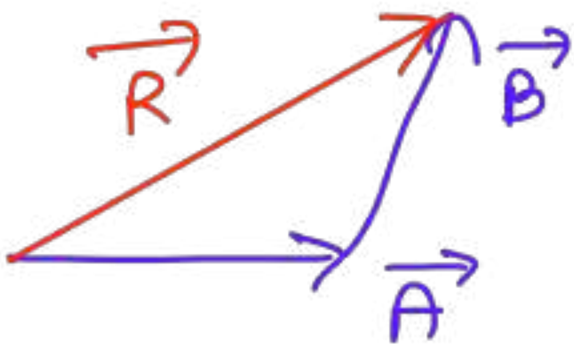


① Graphical method :-

ⓐ Triangle method

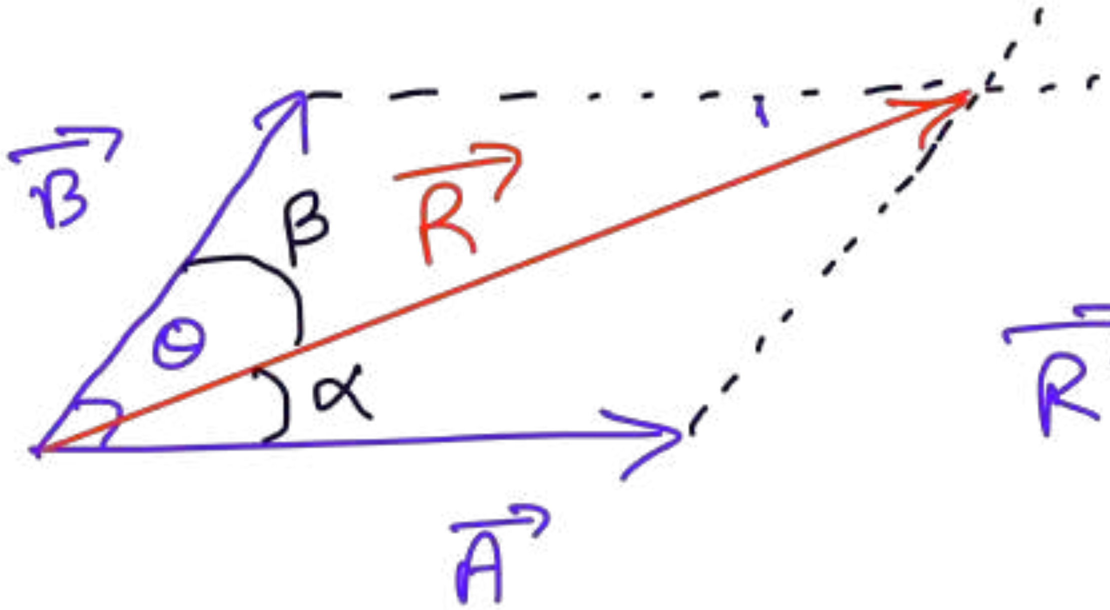
$$\vec{R} = \vec{A} + \vec{B}$$



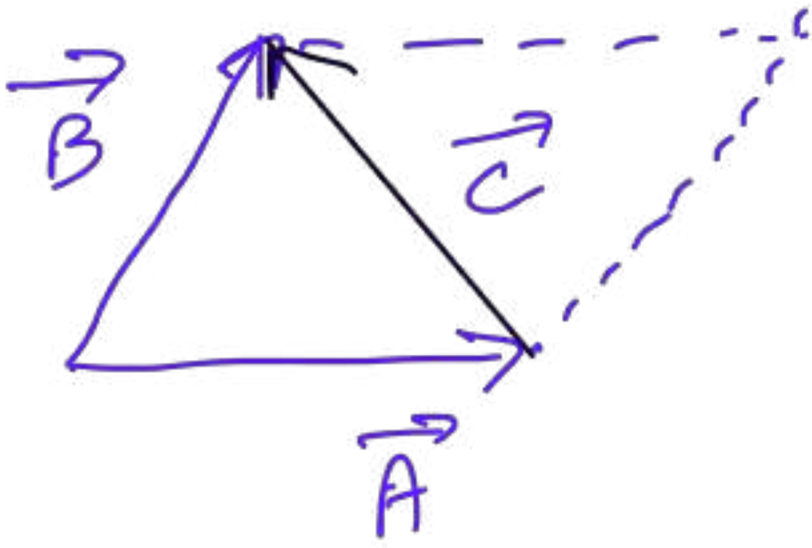


* $\vec{R}$ is in opposite sense of $\vec{A}$ & $\vec{B}$.

(b) Parallelogram method :-



$$\vec{R} = \vec{A} + \vec{B}$$



$$\vec{B} = \vec{C} + \vec{A}$$

$$\vec{C} = \vec{B} - \vec{A}$$

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

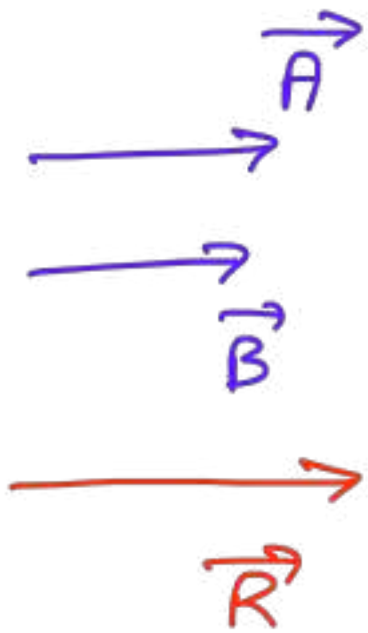
$$\boxed{\beta = \theta - \alpha}$$

Special cases:-

(1) If $\vec{A} \parallel \vec{B}$, $\theta = 0^\circ$

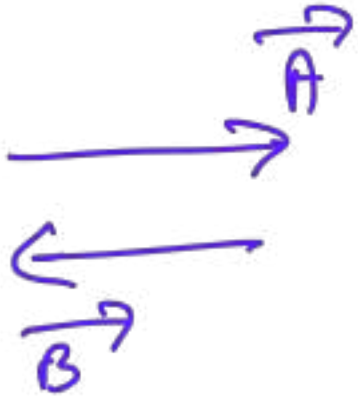
Then $R = A + B$

$\alpha = 0^\circ$



(2) If $\vec{A} \uparrow\downarrow \vec{B}$ (antiparallel)

$$\theta = 180^\circ$$



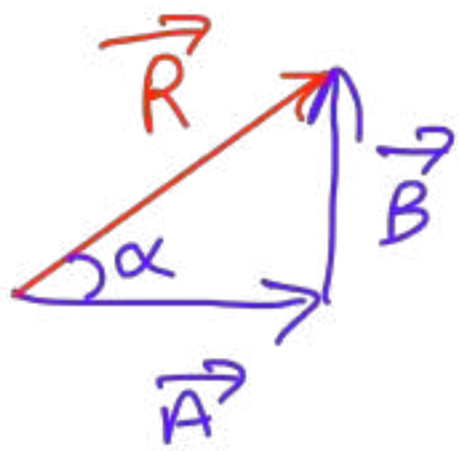
Then $R = |A - B|$

$$\alpha = 0^\circ \text{ or } 180^\circ$$

(3) If $\vec{A} \perp \vec{B}$, $\theta = 90^\circ$

Then $R = \sqrt{A^2 + B^2}$

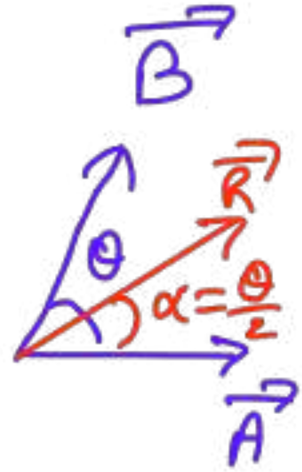
$$\tan\alpha = \frac{B}{A}$$



(4) If $A = B$

Then $R = 2A\cos\left(\frac{\theta}{2}\right)$

$\alpha = \frac{\theta}{2}$

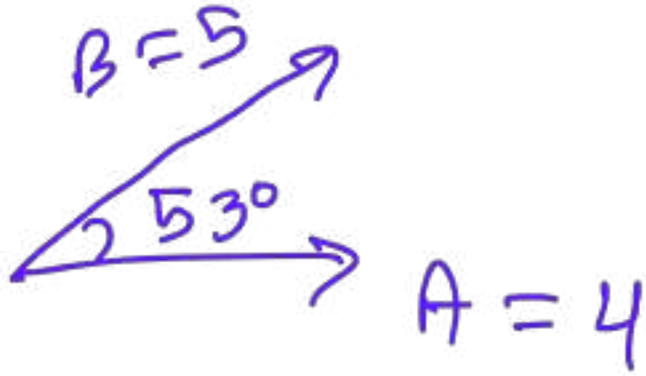


(5) If $A = B$ & $\theta = 120^\circ$

Then $R = A = B$

$\alpha = 60^\circ$

Eg



find R, α.

Sol^n

$$R = \sqrt{4^2+5^2+2\times4\times5\cos53^\circ}$$

$$= \sqrt{24+41}$$

$$= \sqrt{65}$$

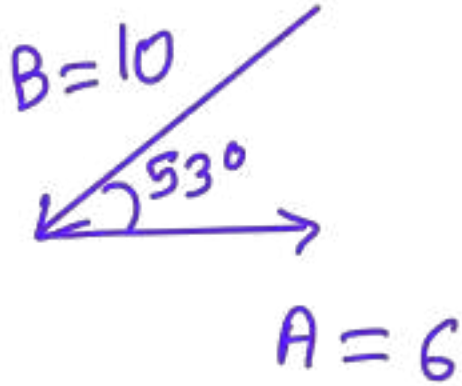
$$\tan\alpha = \frac{5\sin53^\circ}{4+5\cos53^\circ}$$

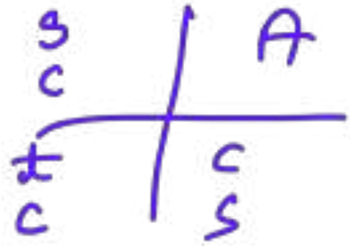
$$= \frac{\cancel{5}\times\frac{4}{\cancel{5}}}{4+\cancel{5}\times\frac{3}{\cancel{5}}} = \frac{4}{7}$$

$$\alpha = \tan^{-1}\left(\frac{4}{7}\right)$$

Eg

find R, α





Soln

$$R = \sqrt{6^2 + 10^2 + 2 \times 6 \times 10 \cos(180 - 53°)}$$

$$R = \sqrt{36 + 100 - 120 \cos 53°}$$

$$= \sqrt{136 - \cancel{120}^{24} \times \frac{3}{\cancel{5}}}$$

$$= 8$$

$$\begin{array}{r} 136 \\ 72 \\ \hline 64 \end{array}$$

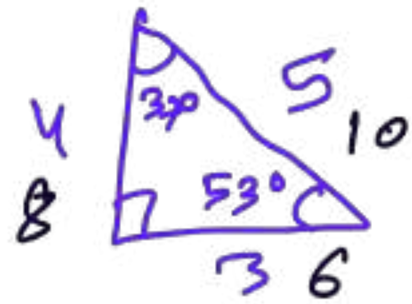
$$\tan\alpha = \frac{10 \sin(180 - 53°)}{6 + 10 \cos(180° - 53°)}$$

$$= \frac{10 \times \sin 53°}{6 - 10 \cos 53°} = \frac{\cancel{10}^{2} \times \frac{4}{\cancel{5}}}{6 - \cancel{10}_{2} \times \frac{3}{\cancel{5}}}$$

$$= \infty \qquad \boxed{\alpha = 90°}$$

3, 4, 5

6 8 10 ✓



Eg $A = B = 25$ $\quad \theta = 37°$

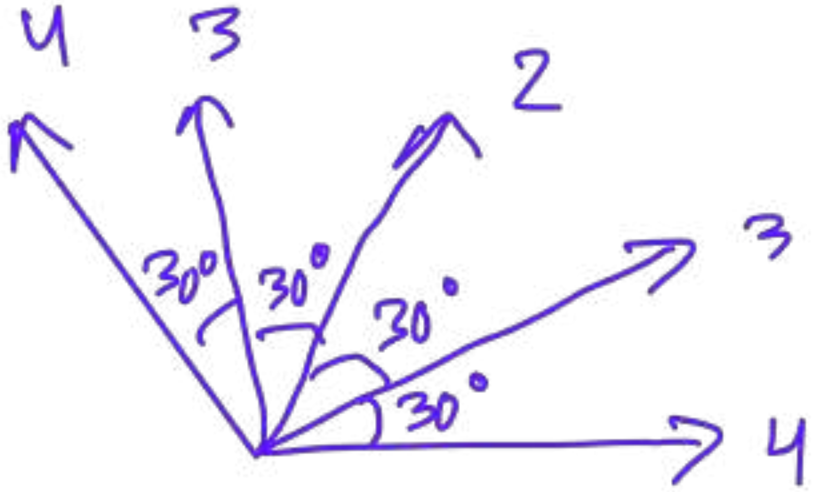
Find R, α

Solⁿ $R = \sqrt{A^2 + B^2 + 2AB\cos 37°}$

$= \sqrt{25^2 + 25^2 + 2 \times 25^2 \times \frac{4}{5}}$

$=$

Eg



Find resultant

Sol^n

$$R_4 = 2 \times 4 \times \cos\left(\frac{120^\circ}{2}\right)$$

$$= 4$$

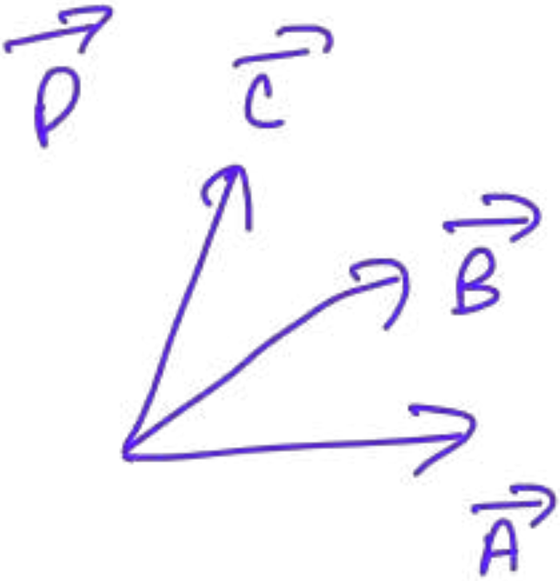
$$R_3 = 2 \times 3 \times \cos\left(\frac{60^\circ}{2}\right)$$

$$= 3\sqrt{3}$$

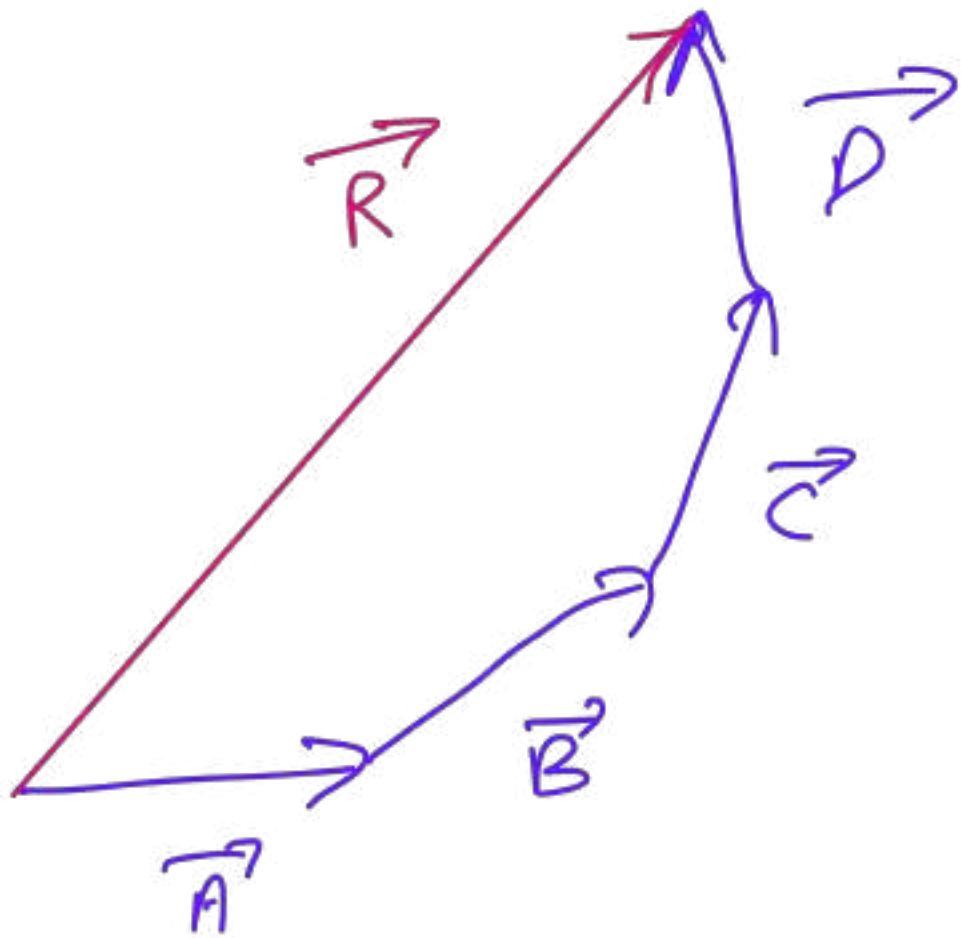
$$R = 2 + 4 + 3\sqrt{3}$$

$$= 6 + 3\sqrt{3}$$

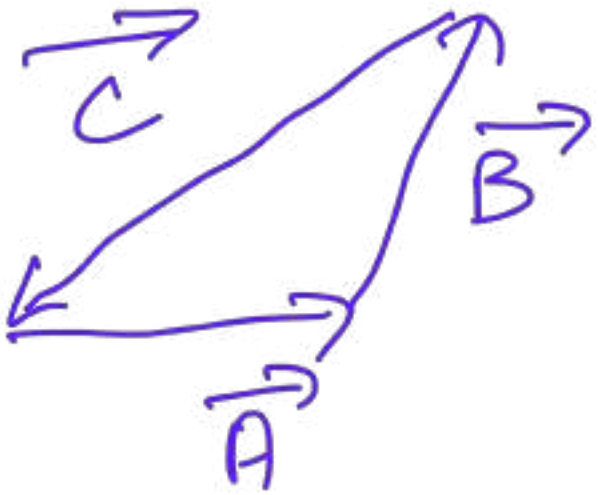
Addition of more than two vectors (Polygon law):-



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

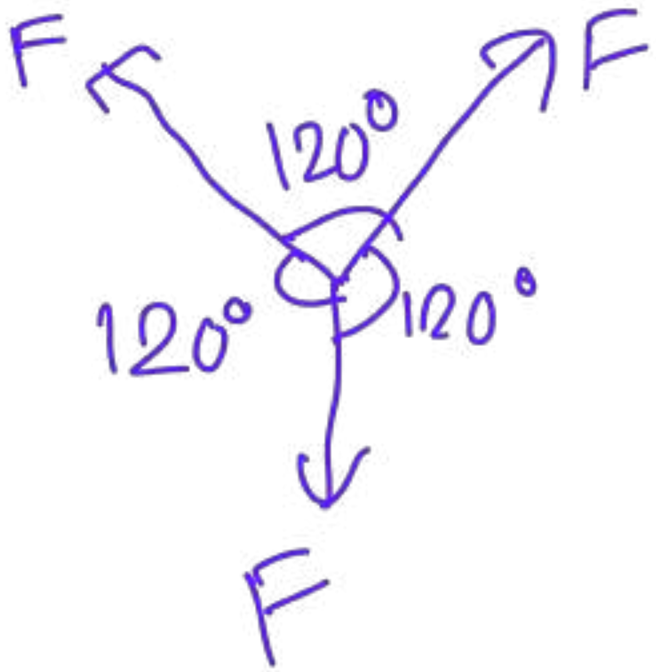


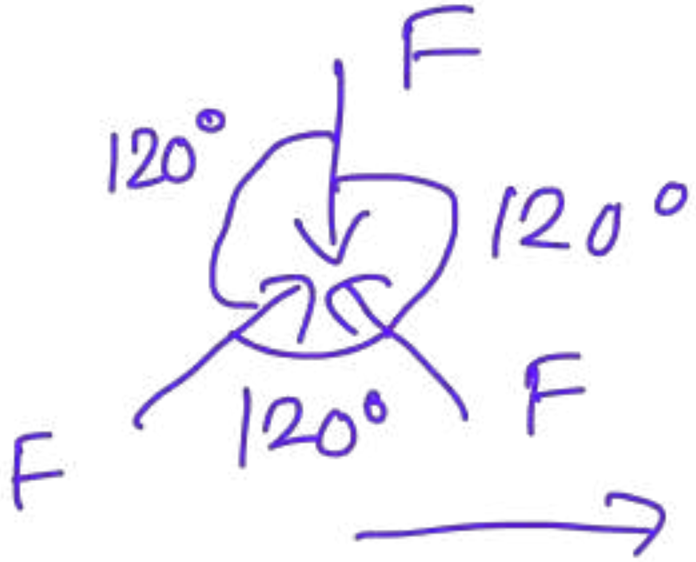
* If vectors can be arranged in form of sides of close polygon in same order then their resultant will be zero.



$$\vec{A} + \vec{B} + \vec{C} = \vec{0}$$

* If N vectors having equal magnitude are arranged at equal mutual angle $\left(\theta = \frac{360^\circ}{N}\right)$ Then their resultant will be zero

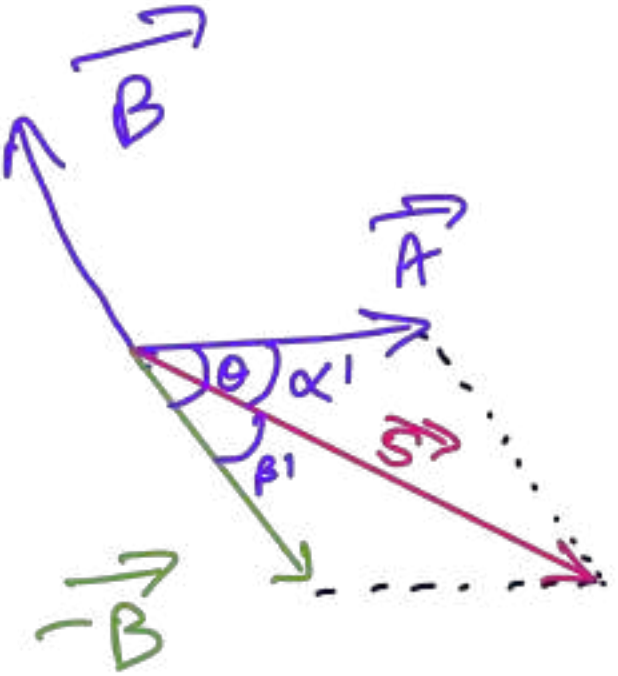




$\vec{R} = \vec{0}$

$R = 0$

Subtraction



$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{S} = \vec{A} - \vec{B}$$

$$= \vec{A} + (-\vec{B})$$

$$\boxed{S = \sqrt{A^2 + B^2 - 2AB\cos\theta}}$$

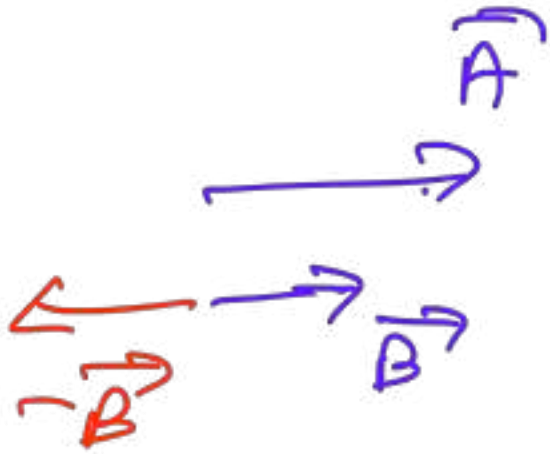
$$\boxed{\tan\alpha' = \frac{B\sin\theta}{A - B\cos\theta}}$$

Special cases:-

① If $\vec{A} \parallel \vec{B}$, $\theta = 0^\circ$

Then $S = |A - B|$

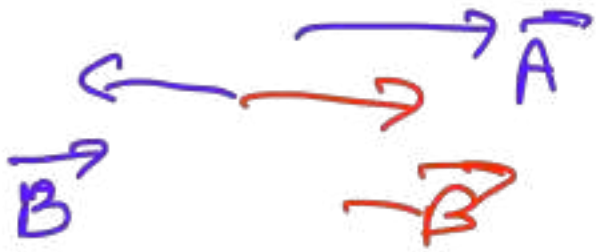
$\alpha' = 0^\circ$ or 180°



② If $\vec{A} \uparrow\downarrow \vec{B}$, $\theta = 180^\circ$

Then $S = A + B$

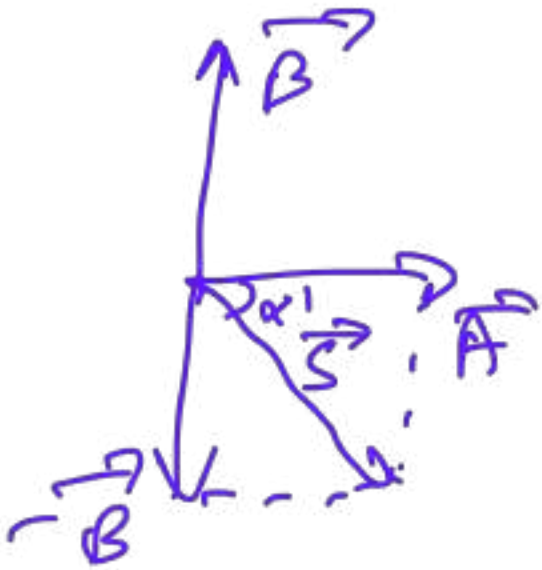
$\alpha' = 0^\circ$

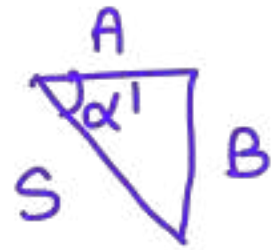


(3) If $\vec{A} \perp \vec{B}$, $\theta = 90°$

Then $S = \sqrt{A^2 + B^2}$

$\tan\alpha' = \frac{B}{A}$

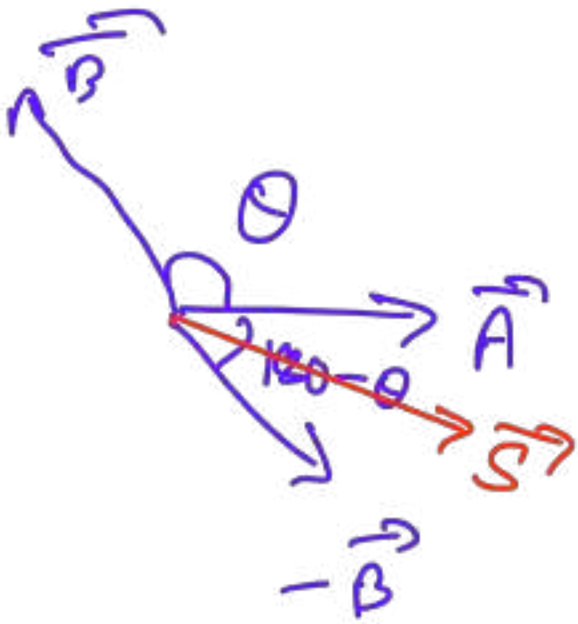




(4) If $A = B$

Then $S = 2A\sin\left(\frac{\theta}{2}\right)$

$\alpha' = \frac{180-\theta}{2} = 90 - \frac{\theta}{2}$



(5) If $A = B$, $\theta = 60^\circ$

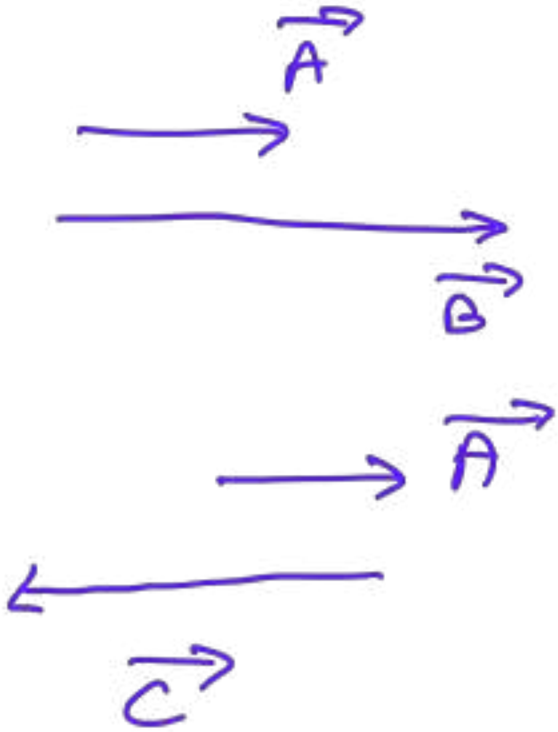
Then $S = A = B$

$\alpha' = 60^\circ$

$$2\vec{A} = \vec{B}$$

$$|\vec{B}| = 2\,|\vec{A}|$$

$$\hat{B} = \hat{A}$$



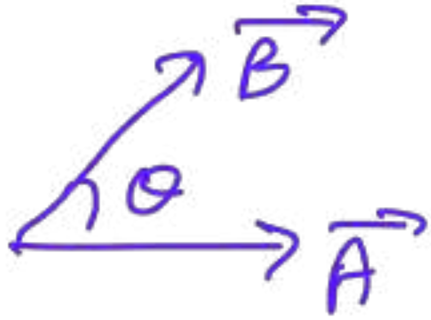
$$\vec{C} = -2\vec{A}$$

$$|\vec{C}| = 2\,|\vec{A}|$$

$$\hat{C} = -\hat{A}$$

Dot product / Scalar product :-

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$



If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y B_y + A_z B_z$$

Eg $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

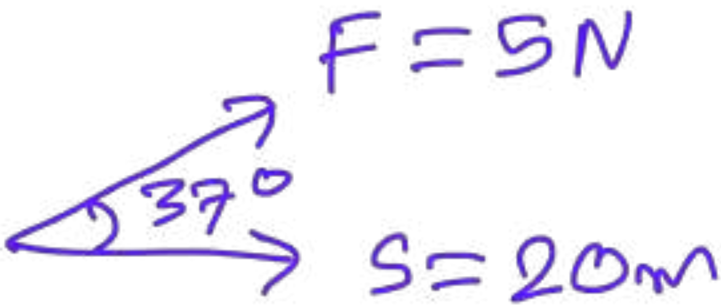
$\vec{B} = -2\hat{i} + 3\hat{j} + 4\hat{k}$

Find $\vec{A} \cdot \vec{B}$

Sol^n $\vec{A} \cdot \vec{B} = 2 \times (-2) + (-3) \cdot (3) + 4 \times 4$

$= -4 - 9 + 16 = 3$

Eg



Find W.

Sol^n

$$W = FS\cos\theta$$

$$= 5 \times 20 \times \frac{4}{5} = 80\,J$$

* If $\vec{A}$ & $\vec{B}$ are orthogonal vectors or normal vectors or perpendicular vectors then

$$\boxed{\vec{A} \cdot \vec{B} = 0}$$

* $\vec{A} \cdot \vec{A} = A^2$

* $-AB \leq \vec{A} \cdot \vec{B} \leq AB$

Applications of dot product :-

① $\boxed{\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{A\ B}}$

Eg. Find angle b/w $3\hat{i} - 4\hat{j} + 5\hat{k}$ and $2\hat{i} + 3\hat{j} - 2\hat{k}$

Solⁿ

$$\cos\theta = \frac{-16}{\sqrt{(3^2+4^2+5^2)}\sqrt{2^2+3^2+2^2}}$$

$$= \frac{-16}{\sqrt{50\times17}}$$

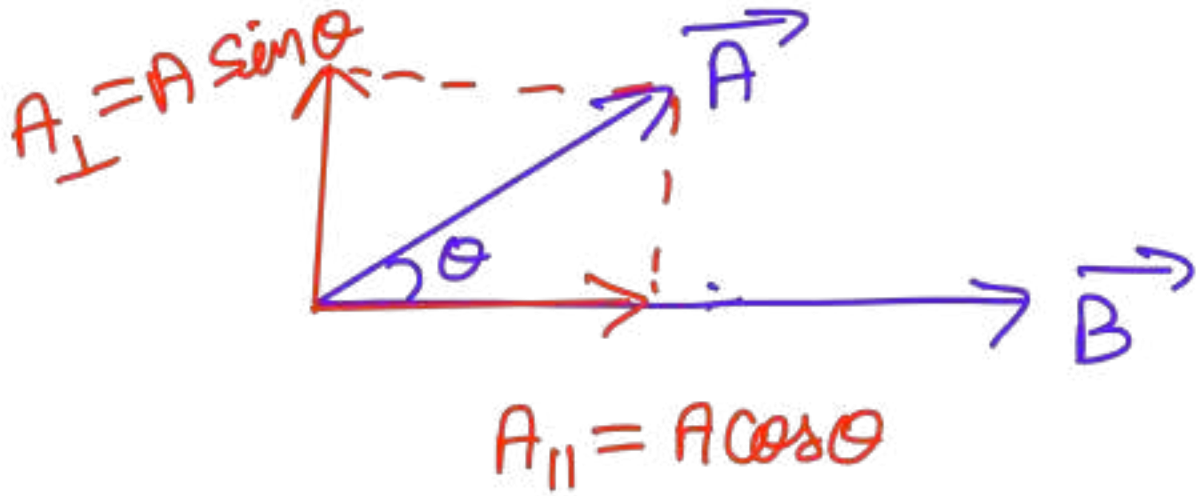
$$\cos\theta = \frac{-16}{5\sqrt{34}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{-16}{5\sqrt{34}}\right)}$$

(2) component of $\vec{A}$ in direction of $\vec{B}$

Or

projection of $\vec{A}$ on $\vec{B}$



(1) Scalar form :-

$$A_{\parallel} = A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B}$$

$$A_\perp = A\sin\theta$$

$$A_\perp = \sqrt{A^2 - A_{\parallel}^2}$$

(2) Vector form :-

$$\vec{A_{\parallel}} = (A\cos\theta)\hat{B} = \left(\frac{\vec{A}\cdot\vec{B}}{B^2}\right)\vec{B}$$

$$\vec{A_{\perp}} = \vec{A} - \vec{A_{\parallel}}$$

Eg $\vec{A} = 2\hat{i} - 3\hat{j} + P\hat{k}$

$\vec{B} = 3P\hat{i} - 2\hat{j} + 6\hat{k}$

If $\vec{A} \perp \vec{B}$ then find P

Soln $\vec{A}\cdot\vec{B} = 6P + 6 + 6P = 0$

$12P = -6 \Rightarrow \boxed{P = -\frac{1}{2}}$

Eg Find component of $2\hat{i} - 2\hat{j} + \hat{k}$ in direction of $12\hat{i} - 3\hat{j} + 4\hat{k}$ and perpendicular to it.

Sol^n $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\vec{B} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{A_{\parallel}} = \left(\frac{\vec{A} \cdot \vec{B}}{B^2}\right)\vec{B}$$

$$= \frac{34}{(12^2 + 3^2 + 4^2)}(12\hat{i} - 3\hat{j} + 4\hat{k})$$

$$= \frac{34}{169}(12\hat{i} - 3\hat{j} + 4\hat{k})$$

$$\vec{A}_{\perp} = \vec{A} - \vec{A}_{\parallel}$$

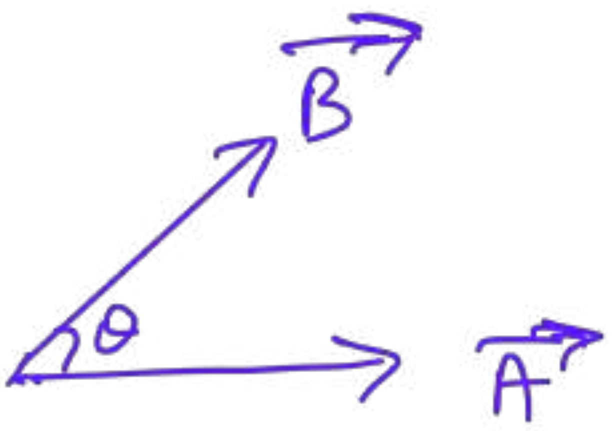
$$= \left(2\hat{i} - 2\hat{j} + \hat{k}\right) - \left(\frac{34}{169}\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right.\right.$$

Cross product / Vector product :-

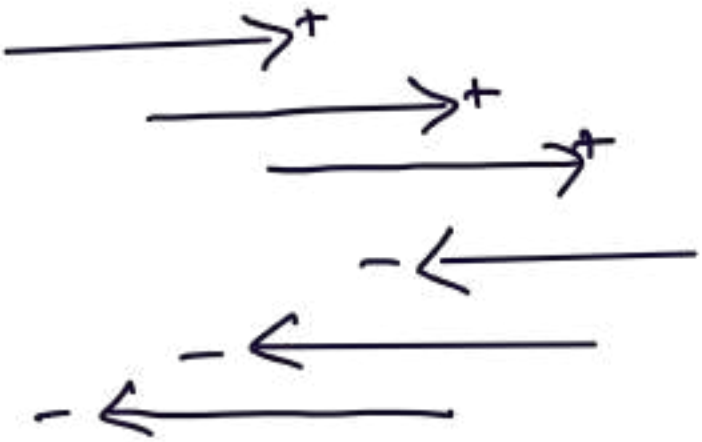
$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

* direction of $\vec{A} \times \vec{B}$ is taken from right hand thumb rule.



$\hat{i} \quad \hat{j} \quad \hat{k} \quad \hat{i} \quad \hat{j}$



If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

Then $\vec{A} \times \vec{B} =$

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\vec{A}$	A_x	A_y	A_z
$\vec{B}$	B_x	B_y	B_z

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix}$$

$$+\hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}\left(A_yB_z - B_yA_z\right) - \hat{j}\left(A_xB_z - B_xA_z\right) + \hat{k}\left(A_xB_y - B_xA_y\right)$$

<u>Eg</u> $\vec{A} = 2\hat{i} - 4\hat{j} - 5\hat{k}$

$\vec{B} = \hat{i} - 3\hat{j} + 2\hat{k}$

Find $\vec{B} \times \vec{A}$

<u>Sol^n</u>

$$\begin{array}{c} \\ B \\ A \end{array}\begin{vmatrix} i & j & k \\ 2 & -4 & -5 \\ 1 & -3 & 2 \end{vmatrix}$$

$$\vec{B}\times\vec{A} = \hat{i}(-8-15) - \hat{j}(4-(-5))$$
$$+\hat{k}(-6-(-4))$$
$$= -23\hat{i} - 9\hat{j} - 2\hat{k}$$

Eg $\vec{A} = 2\hat{i} - 5\hat{j} - \hat{k}$

$\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{A}\times\vec{B} = ?$

Soln

$$\vec{A}\times\vec{B} = \hat{i}(-15-2) - \hat{j}(6-(-1))$$
$$+\hat{k}(-4-(-5))$$
$$= -17\hat{i} - 7\hat{j} + \hat{k}$$

* If $\vec{A} \parallel \vec{B}$ or $\vec{A} \uparrow\downarrow \vec{B}$ or $\vec{A}$ & $\vec{B}$ are collinear vectors Then $\boxed{\vec{A} \times \vec{B} = \vec{0}}$

* $\vec{A} \times \vec{A} = \vec{0}$

* $0 \leq |\vec{A} \times \vec{B}| \leq AB$

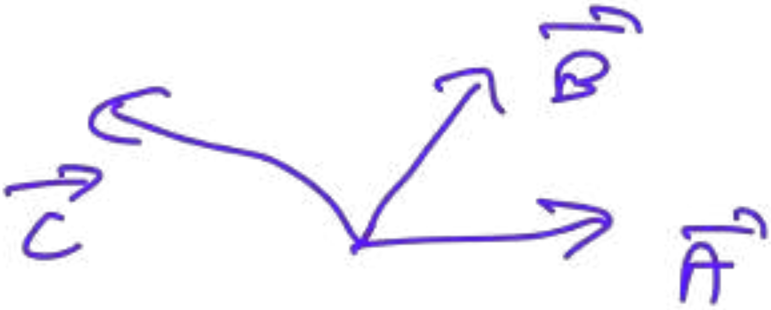
* $\vec{B} \times \vec{A} = -(\vec{A} \times \vec{B})$

Applications of vector product :-

① Condition for coplanarity :-

If $\vec{A}$, $\vec{B}$ & $\vec{C}$ are coplanar then

$$\boxed{(\vec{A} \times \vec{B}) \cdot \vec{C} = 0}$$

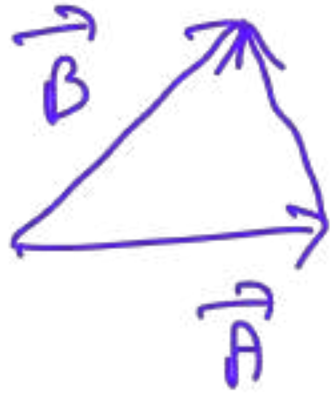


② To calculate area :-

ⓐ Area of triangle :-

If $\vec{A}$ & $\vec{B}$ are sides of triangle then

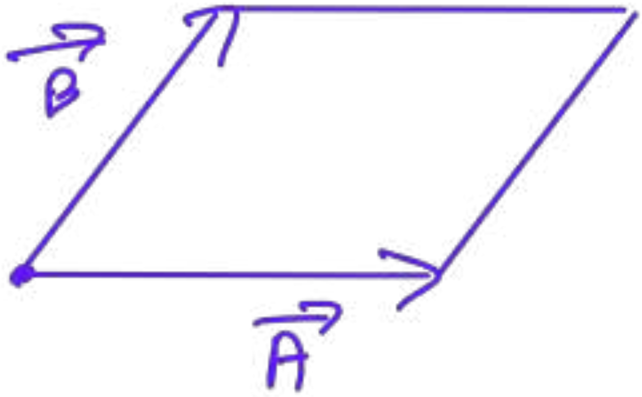
$$\boxed{\text{Area} = \frac{1}{2}|\vec{A} \times \vec{B}|}$$



ⓑ Area of parallelogram

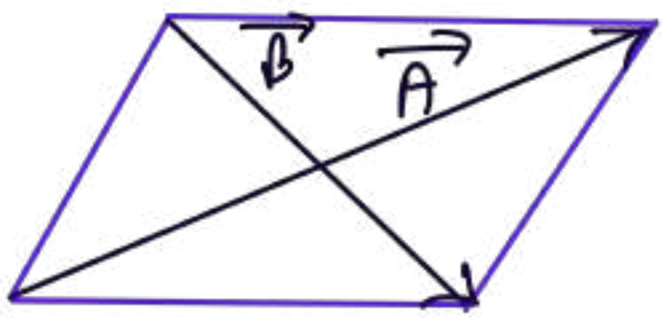
*If $\vec{A}$ & $\vec{B}$ are adjacent sides of parallelogram then

$$\text{Area} = |\vec{A} \times \vec{B}|$$



* If $\vec{A}$ and $\vec{B}$ are diagonals of parallelogram then

$$\boxed{\text{Area} = \frac{1}{2}|\vec{A} \times \vec{B}|}$$



(3) Various formulae:-

Eg $\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{\tau} = \vec{r} \times \vec{F}$

Eg If $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{B} = 6\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{C} = \underline{2\hat{i} - 3\hat{j} + p\hat{k}}$ ✓

are coplanar then find p

Soln $(\vec{A} \times \vec{B}) \cdot \vec{C} = 0$

$$\vec{A} \times \vec{B} = 4\hat{i} - 32\hat{j} - 22\hat{k}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = 8 + 96 - 22p = 0$$

$$p = \frac{104}{22} = \frac{52}{11}$$

* If $\vec{A} \parallel \vec{B}$ then

$$\boxed{\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}}$$

Eg $\vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{B} = 2\hat{i} + P\hat{j} - q\hat{k}$

Find P & q if $\vec{A} \parallel \vec{B}$

Solⁿ

$$\frac{3}{2} = \frac{-2}{P} = \frac{4}{-q}$$

$$\boxed{P = \frac{-4}{3}} \qquad \boxed{q = \frac{-8}{3}}$$